08/03/18

- if S_1, then S_2.

i.e. $S_1 \Rightarrow S_2$ "S_1 implies S_2".

Truth Table:

S_1	S_2	$S_1 \Rightarrow S_2$
T	T	T
T	F	F
F	T	T
F	F	T

only False when S_1 is true and S_2 is false
if ~~S_2~~ S_1 is false then S_2 could be anything, so true.

- S_1 iff S_2 (if and only if)

i.e $S_1 \Leftrightarrow S_2$

means $(S_1 \Rightarrow S_2)$ and $(S_2 \Rightarrow S_1)$

Truth Table

S_1	S_2	$S_1 \Leftrightarrow S_2$
T	T	T
T	F	F
F	T	F
F	F	T

(like XNOR)

eg: show that $S_1 \Leftrightarrow S_2 \equiv (S_1 \Rightarrow S_2)$ AND $(S_2 \Rightarrow S_1)$

$\equiv$ equivalent

i.e. show that they have same truth table.

S_1	S_2	$S_1 \Rightarrow S_2$	$S_2 \Rightarrow S_1$	$(S_1 \Rightarrow S_2)$ and $(S_2 \Rightarrow S_1)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

same as TT for $S_1 \Leftrightarrow S_2$

Notations:

- $S_1 \Rightarrow S_2$ (if S_1 then S_2) (S_1 implies S_2)
- $S_1 \vee S_2$ (S_1 or S_2)
- $S_1 \wedge S_2$ (S_1 and S_2)
- $S_1 \oplus S_2$ (exclusive or) (either S_1 or S_2 but not both).

Truth table for $S_1 \oplus S_2$:

S_1	S_2	$S_1 \oplus S_2$
T	T	F
T	F	T
F	T	T
F	F	F

in Boolean Algebra, T = 1 and F = 0.

- $\neg S_1 = S_1'$ (negation / complement)

Truth table

S	$\neg S$
1	0
0	1

conclude: $S_1 \Leftrightarrow S_2 \equiv (S_1 \oplus S_2)' \equiv \neg(S_1 \oplus S_2)$

eg: $\underbrace{x^2+1=0}_{S_1}$ iff $\underbrace{x^2+3=1}_{S_2}$ for some $x \in \mathbb{R}$

$$(\underbrace{S_1 \Rightarrow S_2}_{T} \wedge \underbrace{S_2 \Rightarrow S_1}_{T}) = T$$

Hence $S_1 \Leftrightarrow S_2$ is true

String of 0's and 1's.

eg: • $(010111) \vee (110110)$ • $(0010) \wedge (1011)$

$$\begin{array}{r} 010111 \\ \vee\ 110110 \\ \hline 110111 \end{array} \qquad \begin{array}{r} 0010 \\ \wedge\ 1011 \\ \hline 0010 \end{array}$$

• $(1100) \oplus (1101)$

$$\begin{array}{r} 1100 \\ \oplus\ 1101 \\ \hline 0001 \end{array}$$

Note: $(S_1 \oplus S_2)' \equiv (S_1 \wedge S_2) \vee (S_1' \wedge S_2')$

Sets:

$A = \{3, x, 4, \{x, 2\}, 7\}, 2\}$

$\subset \rightarrow$ compare between 2 sets (subsets).

$\in \rightarrow$ betw. element and a set. (belong).

$x \in A$ "x belongs to A"
"x is an element of A"

$\{x, 2\} \in A$ "$\{x, 2\}$ is an element of A"

$7 \in A$ "7 is an element of A".

$\{4, x\} \subset A$ "set of 2 elements, 4 and x, is a subset of A".

Eg: $A = \{\{3,2\}, x, \{x\}, 3, 2, \phi\}$.

$\{x\} \in A$, $\{x\}$ is an element of A. Things on the left must be exactly inside A. TRUE

$\phi \in A$, TRUE.

$\{2, x\} \in A$. FALSE

$\boxed{\text{by default: } \phi \subset \text{any set.}}$

$\{2,3\} \in A$. TRUE
$x^2 \in A$. FALSE

$\{2,3\} \subset A$. $\{2,3\}$ is a subset of A.
i.e Does $2 \in A$? Yes
Does $3 \in A$? Yes
Thing on left must be in set notation $\{\ \}$ $A = \{\{3,2\}, x, \{x\}, \underline{3}, \underline{2}, \phi\}$
TRUE.

$\{\{3,2\}, 2, 3\} \subset A$. TRUE. $A = \{\underline{\{3,2\}}, x, \{x\}, \underline{3}, \underline{2}, \phi\}$
* each element in LHS must be $\in A$

$\{\phi, 2\} \subset A$. TRUE.

$\{\{x,2\}, 3\} \subset A$. FALSE bc $\{x,2\} \notin A$.

eg: $4 \in \mathbb{Z}$. TRUE $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

$\{4\} \subset \mathbb{Z}$. TRUE.

$\phi \in \mathbb{Z}$. ~~TRU~~ FALSE. ϕ (empty set) not an element.
$\phi \subset \mathbb{Z}$. TRUE. bc. $\phi \subset$ any set.

$A = \{2, 3, \{5\}, 7, \{5,2\}\}$ $B = \{5, 2, \{3,7\}, \phi\}$.

$A \cup B$ (union) → similar to OR $\vee$.
$A \cap B$ (intersection) → similar to AND $\wedge$

Note. Don't repeat elements in set.
$\{1, 1, 2\}$ ✗.
$\{1, 2\}$ ✓

$A \cup B = \{2, 3, \{5\}, 7, \{5,2\}, 5, \{3,7\}, \phi\}$.

$A \cap B = \{2\}$. always use set notation.

$A - B$ = set of elements in A but ~~bu~~ not in B (difference betw. A and B.
$= \{3, \{5\}, 7, \{5,2\}\}$

$B - A = \{5, \{3,7\}, \phi\}$. Note $A - B \neq B - A$

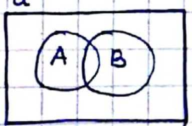
Notations: $\overline{X} = X' = c(X)$ = complement of X

Universal set.

Universal set $= U$
↳ our planet

$A \subset U$, $B \subset U$. Diagram:

U



Assume $U = \{2, 3, \{5\}, 7, \{5,2\}, 5, \{3,7\}, \phi, \mathbb{Z}, \{\theta, 2\}, \{7, \mathbb{Z}\}, 22, 0\}$.

$\overline{A} = U - A$ = set of elements in U but not in A.
$\overline{A} = \{\{3,7\}, \phi, \mathbb{Z}, \{\theta, 2\}, \{7, \mathbb{Z}\}, 22, 0\}$

Diagram :

$\rightarrow \bar{A}$

$\{\theta, 2\} \in U$. TRUE

$\{\theta, 2\} \subset U$. TRUE

$\{\{5,2\}\} \in U$. FALSE (extra brackets)

$\{\{5,2\}\} \subset U$. TRUE

element

set.

Note : $\{2, x\} = \{x, 2\}$.
Order not important.

Definition : A, B are sets. Cartesian - product of A, B:
$= A \times B = \{(a,b) \mid a \in A, b \in B\}$.
Set of all ordered pairs (a,b) where 1st co-ordinate comes from A , 2nd co-ordinate comes from B.

$A = \{2, x\}$ $B = \{y, 3, 1\}$
Find $A \times B = \{(2,y), (2,3), (2,1), (x,y), (x,3), (x,1)\}$
- $(2,y) \in A \times B$. TRUE
- $(y,2) \in A \times B$. FALSE bc cartesian - product.
- $\{(2,3), (y,x)\} \subset A \times B$. FALSE bc $(y,x) \notin A \times B$
  $(x,y) \in A \times B$.

Assume $|A| = n$ (cordinality of set = size = order)
Assume $|B| = m$.
Then $|A \times B| = nm$.

Homework

1 (i) Use truth table to convince me that $(S_1 \Rightarrow S_2) \vee (S_1 \Rightarrow 3) \equiv S_1 \Rightarrow (S_2 \vee S_3)$

ans :

S_1	S_2	S_3	$S_1 \Rightarrow S_2$	$S_1 \Rightarrow S_3$	$(S_1 \Rightarrow S_2) \vee (S_1 \Rightarrow S_3)$	$S_1 \Rightarrow (S_2 \vee S_3)$
1	1	1	1	1	1	1
1	1	0	1	0	1	1
1	0	1	0	1	1	1
1	0	0	0	0	0	0
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

Hence, $(S_1 \Rightarrow S_2) \vee (S_1 \Rightarrow S_3) \equiv S_1 \Rightarrow (S_2 \vee S_3)$

ii) Use the truth table to convince me that $S_1 \wedge S_2' \equiv (S_1' \vee S_2)'$

S_1'	S_1	S_2	S_2'	$S_1 \wedge S_2'$	$(S_1' \vee S_2)$	$(S_1' \vee S_2)'$
0	1	1	0	0	1	0
0	1	0	1	1	0	1
1	0	1	0	0	1	0
1	0	0	1	0	1	0

Hence $S_1 \wedge S_2' \equiv (S_1' \vee S_2)'$

iii) Let $A = \{0, \{0, y\}, y, \{6\}, 6, x, \phi\}$ $\quad B = \{\{0\}, \{\phi\}, \{6\}, \{6, x\}, 6, y, 23, 10, \{\{0\}, \{6, x\}\}\}$.

Write T or F.

a) $\{\{0\}, \{6, x\}\} \in B \rightarrow T.$

b) $\{\underbrace{\{0\}}, \underbrace{\{6, x\}}\} \subset B$

elements.

Check $\{0\} \in B$ ✓

$\{6, x\} \in B$ ✓

$\therefore \{\{0\}, \{6, x\}\} \subset B \rightarrow T.$

c) $\{\phi\} \in A \rightarrow F$

d) $\{\phi\} \in B \rightarrow T$

e) $\{\phi\} \subset B$

check. $\phi \in B$ ✗

$\therefore \{\phi\} \subset B \rightarrow F$

f) $\{\phi\} \subset A$

Check $\phi \in A$ ✓

$\therefore \{\phi\} \subset A \rightarrow T$

g) $\phi \in A \rightarrow T$

h) $\{23, 10, y\} \in B \rightarrow F$

i) $\{23, 10, y\} \subset B$

Check $23 \in B$ ✓ $\quad 10 \in B$ ✓ $\quad y \in B$ ✓

$\therefore \{23, 10, y\} \subset B \rightarrow T$

j) $\{6\} \in A \cap B$

First: $A \cap B = \{y, \{6\}, 6\}$

$\therefore \{6\} \in A \cap B \rightarrow T.$

k) $\{6\} \subset A \cap B$

check $6 \in A \cap B \rightarrow$ ✓

$\therefore \{6\} \subset A \cap B \rightarrow T$

l) $(10, x) \in \cancel{A \cap B}$ $A \times B$

10 should be from set A. ✗

$x \in B$. ✗

$\therefore (10, x) \in A \times B \rightarrow F$

m) $\{\underbrace{(\{0, y\}, 6)}_{\text{element}}, \underbrace{(y, \{0\})}_{\text{element}}\} \subset A \times B$

$\{0, y\} \in A$ ✓ $\quad y \in A$ ✓

$6 \in B$ ✓ $\quad \{0\} \in B$ ✓

$\therefore \{(\{0, y\}, 6), (y, \{0\})\} \subset A \times B \rightarrow T$

n) Find $A \cap B$.
$A \cap B = \{ y, \{6\}, 6 \}$.

o) Find $B - A$ i.e elements in B that are not in A.
$B - A = \{ \{0\}, \{\phi\}, \{6, x\}, 23, 10, \{ \{0\}, \{6, x\} \} \}$.

p) Find $|A \times B|$. i.e find the cardinality of the cartesian product $A \times B$
$|A| = 7$
$|B| = 9$
$|A \times B| = 63$.

13/03/18 A is a set. $|A| = n < \infty$
$P(A)$ = power set = { all subsets of A }, $|P(A)| = 2^n$

eg: $A = \{a, b\}$, $|A| = 2$
Find $P(A)$, $|P(A)|$.

in every set, the whole set is a subset of itself. and the empty set is always a subset of any set.

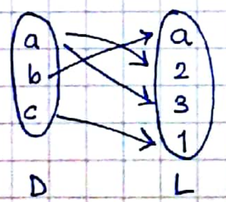
$P(A) = \{ \{a, b\}, \phi, \{a\}, \{b\} \}$.

Statements: $\{a\} \in P(A) \rightarrow T$
$a \in P(A) \rightarrow F$
$a \in A \rightarrow T$
$\{ \{a\}, \{b\} \} \subset P(A) \rightarrow T$
$\{ \phi, \{a\} \} \subset P(A) \rightarrow T$
$\phi \in P(A) \rightarrow T$
$\phi \subset P(A) \rightarrow T$ by default.
$\{\phi\} \subset P(A) \rightarrow T$

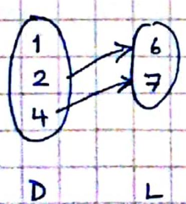
eg: $A = \{a, b, 3\}$, $|A| = 3$
$|P(A)| = 8$
$P(A) = \{ A, \phi, \{a\}, \{b\}, \{3\}, \{a, b\}, \{a, 3\}, \{b, 3\} \}$. *remember, order does not matter.

Statements: $A \in P(A) \rightarrow T$
$A \subset P(A) \rightarrow F$
$\{A\} \subset P(A) \rightarrow T$

Definition: a function, $f : D \rightarrow L$, is a mapping from one set (D) to another set (L), where D is domain and L is co-domain, s.t. each element in the domain corresponds to one and only one element in the co-domain.

eg


not a function.

eg


not a function bc. 1 does not correspond to anything